Letter Grading System is fundamentally Unfair by its very Design

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Abstract:

A systems study of the most imposing, impressive and widely prevalent *Letter Grading System* (LGS) is presented. In this first part of the report, it is shown that the LGS system exhibits a chaotic behavior which is unfair to the students, unfair to the teachers, and unfair to all those who need to rely on the information contained in and communicated through those grade reports. The LGS system simply fails to meet the essential purpose for which the *Academic Performance Evaluation of Students* (APES) is undertaken in Academia.

An analysis of its chaotic behavior will be presented as the second part of the report, in a later paper - *Academic Performance Evaluation of Students - Ubiquitous System Analyzed* (APES-USA) - clearly indicating that the root cause is some serious lacunae inherent in its very design.

A proposed solution to this 'APES-USA' problem will be presented as the third part of this report.

Key Words:

Raw-Scores, Letter Grading System, Letter-Grades,
Grade-Point-Average (GPA), Cumulative Grade Point Average (CGPA),
Distribution Based Grading, Distribution Independent Grading,

1. Introduction

The most imposing, impressive and widely prevalent *Letter Grading System* (LGS) model requires each of the teachers as evaluators, to assign an appropriately chosen letter grade (from among the set of assignable letter grades as specified in the academic rules & regulations and as per the guidelines laid out by the school/college/university/institute) to each of the students in each of the courses/subjects/papers and report the same, at the end of every academic session/term/quarter/semester. For this purpose, the teacher may be required to conduct a sequence of quizzes/tests/examinations/etc., (an almost continuous, incremental as well as cumulative evaluations) that together facilitate in assessing/measuring the extent/degree of proficiency achieved by the student in the subject, based on the performance of the student in such tests etc., which when well designed would evenly cover the entire subject material that a student is expected to master during that period of study.

This paper presents the first part of the report of a detailed study of the existing LGS system for the Academic Performance Evaluation of Students.

It is shown that the LGS system exhibits a chaotic (unpredictable although deterministic), highly complex, counter-intuitive as well as undesirable behavior. Specifically, three kinds of chaotic system-intrinsic phenomena have been identified: Chaotic System Biased Amplification or Attenuation of Differentials (CSBAAD), Chaotic System Biased Suppression or Expression of Differentials (CSBSED), Chaotic System Biased Relative Rank Inversion (CSBRRI). These system-intrinsic phenomena of Chaotic System Biased Information Loss/Corruption (CSBILC) result in Chaotic System Biased Unfair and Unreliable Comparisons (CSBUUC) among the students being evaluated, making it to be very grossly unfair to the students. It is also unfair to the prospective recruiters or employers who expect some relevant, unbiased, reliable information to be contained in and conveyed through those transcripts or grade reports. Again, it is very unfair to the teachers, who find themselves utterly helpless, when the Raw Scores/Marks that they had originally assigned very conscientiously, are later subjected to chaotic system biased information loss/corruption.

Also, the LGS system design wrongly presumes that the teacher's precision in assessment/evaluation is rather poor, limited to classification into possibly only a handful of distinct categories. However, there is an intrinsic contradiction in the system design philosophy itself since a significantly higher precision level is mysteriously presumed to have been achieved in reporting the final figures of GPA or CGPA, etc. This is like expecting a double precision accuracy in some computational result when all the computations are performed in single precision although the input is in double precision. On the other hand, please note that if in fact the GPA and CGPA are to be reported after duly rounding them off to the grade point values corresponding to the nearest letter grade, then there is nothing much to claim by this study.

The LGS system just simply fails to provide a reliable/precise and robust/resilient mechanism for a true representation and communication of otherwise appropriate/relevant information as to what the teachers really meant to convey, regarding their unbiased/objective and fair/justifiable assessments/evaluations originally conducted.

2. A Typical Letter Grading System (LGS) Model

Although there are several minor variations among the different letter grading systems that are prevalent in the various educational institutions, in terms of the parameters incorporated in such a system design, the essential system behavior observed in these different scenarios happen to be the same! The observed differences and variations among them are only in terms of the nature and/or the extent and/or the locus, of the very same, essentially general, system behavior, which we present here, by taking a typical grading system model, although the same observations/comments can be made with regard to any other specific letter grading system model, belonging to the wide spectrum of various possible distinctly different specific system models.

Let us suppose that the set of valid assignable *Letter Grades* and the associated *Quality/Grade Points* be as follows:

A=5, B=4, C=3, D=2, E=1, F(fail)=0.

Other letter grades (like Pass/Fail, etc) may not carry Quality/Grade Points, but only indicate certain essential information about the Academic Progress of a student towards one's Graduation Requirements. Once a letter grade is assigned to each of the courses that a student had "*registered for credit*", the *Grade Point Average (GPA)* for that student is computed as the weighted average of the *Grade Points* associated with the corresponding letter grades, with the *Course Credits/Units* (CCU) of the corresponding courses as the multiplicative weighting factors for this computation.

Now let us focus on the problem that a typical teacher would have to face, in order to assign an appropriate letter grade to a student on a particular subject. The teacher does go through the evaluation of the answer scripts/papers of a sequence of quizzes/tests/exams/etc, in order to determine the final letter grade, based on some well defined mechanism as per institute academic regulations, usually announced to a class of students at the very beginning of the academic term. For example, a teacher may decide and therefore announce to the students that there would be four one-hour tests each carrying 25% weightage in determining the final grade. Now, how is a test paper evaluated? How, in a given test paper, the answer to each of the questions, evaluated? And how are these combined together (aggregation process/mechanism) to form what we call as the Raw-Score for a test paper?

We believe that no teacher starts off with assigning appropriately chosen letter grades to each of the various questions in a test. Usually, while setting the Question Paper for a test, appropriate numerical marks/scores are assigned for each of the questions, maybe on a Numerical Scale in the range 0-100 (Percentage Scale). While evaluating a test answer paper, appropriate mark/score is given for the answer to each of the questions, based on how close it is to the expected (standard?) answer, or to what extent the answer is acceptable (in case of possible multiple correct/acceptable answers, like in design problems), and these numerical scores are simply added together to come up with a total numerical score, which is the Raw-Score for that particular test paper.

If in fact it be possible, to assign letter grades right at the level of evaluation of each of the questions in a quiz/test/exam, and if in fact there is a well accepted standard procedure designed to somehow combine them into an aggregate letter grade for that particular quiz/test/exam, and then again combine such letter grades for each of the four or more tests/etc into a final letter grade for that student in that specific course; then we do not have much to say in this paper! The whole problem arises because one needs to resort to numbers, in order to have a well defined rational scheme for any measurements, and also to enable us in combining these various scores into a consolidated overall aggregate measure of whichever entity is being measured.

It is quite evident that a teacher has to use some mapping-scheme to convert the raw-scores to the letter-grades (on a grade points scale). This mapping scheme may be either specified by the school or each teacher may be given the flexibility/freedom of designing one's own (although such ad hoc freedom, in itself, without any guidelines thereof, can possibly become an even more serious lacuna in the system). Any specification of a mapping scheme must include an appropriate partitioning of the raw scores scale into a sequence of intervals, so that each interval gets mapped onto its corresponding point in the grade point scale and therefore the corresponding letter grade. Although there can be several specific mapping schemes, they may be anywhere in between the two extreme generic types: (i) distribution

based mapping and (ii) distribution independent mapping; thus giving rise to the corresponding classification of the grading system.

Distribution based grading system can be further classified into two broad categories - one of them may be called '*grading on the curve*' or '*grading based on some chosen standard distribution*' wherein the letter grades are *force-fitted* onto a standard normalized distribution curve, like the Gaussian Probability Distribution Curve. Since the actual distribution of raw scores in a class is (supposed to be somewhat) independent of the teacher and the evaluation system, the only way that the letter grades can be made to fit into a given standard distribution is to have the mapping scheme dependent on the actual distribution of the raw scores in the class; hence this particular name for it. This approach has several proponents and/or followers. However, we believe that *any force-fitting performed on any set of raw data leads to Information Loss/Corruption,* and "we can only end up seeing what we look for". Therefore, for the purpose of having a typical grading system model, we shall not use this scheme. This decision in itself, is not a limitation (of being seemingly dependent on the system model) in terms of the scope of validity or the conclusions that can be drawn from this study, since any mapping scheme belonging to any of these two extreme types, or any other type, generic or otherwise, can be shown to have the *same common system characteristics* which in fact cause the system to exhibit the undesirable behavior that is explained in this paper.

A second category under the generic type of distribution based grading system adapts its mapping scheme dependent upon the actual distribution of the raw scores in a class, although there is no force-fitting of the letter grades (grade points) done to conform to any specific standard normalized distribution curve as such.

Distribution independent grading system uses a fixed mapping scheme that is based on some well defined divisions of the raw scores scale into intervals, the specification of these intervals, and also the associated mapping scheme being independent of the actual distribution of the raw scores in a class.

For the purpose of our study, a distribution independent mapping scheme is considered, and specifically, we shall take it to be as follows:

[100-90] : A; (90-80] : B; (80-70] : C; (70-60] : D; (60-50] : E; (50-00] : F(fail)

This mapping scheme, to convert the *Numerical Raw Scores* to *Letter Grades*, along with the above defined *Grade Points* for each of the letter grades, can be seen to result in an overall computational procedure for our typical system model; in order to determine the letter grade for each of the courses, for each of the students, using the available numerical raw scores; and also to compute the *Grade Point Average (GPA)*, at the end of an academic term/semester; and similarly the Cumulative Grade Point Average (CGPA) for successive academic terms in the program of study. The parameters that define or *characterize* such a mapping scheme from the raw scores to the letter grades, and thus determine the associated computational procedure, include the raw-score interval-domains (that gets mapped onto each of the letter-grades/quality-points) that are non-overlapping and laid out in a well ordered sequence. This characterization is common to any mapping scheme irrespective of whether it belongs to either of the two extremes, namely the "distribution based grading system" or "distribution independent grading system", or even any other possible LGS design. The differences are in terms

of the size and hence the relative positioning of each of the raw-score interval-domains that get mapped onto each of the letter-grades; all other characteristics of the system behavior being essentially same. This justifies the above choice of a simple fixed mapping scheme for the sake of illustrating the system characteristics.

3. Chaotic LGS System Behavior

First, let us make a simple observation regarding the situation encountered in comparing the grades obtained by students in a specific single course. It is not at all uncommon that the raw score of one student will be at the lower end of the interval corresponding to one's grade while that of another will be at the high end of the corresponding interval. For example, consider a situation with four students S1, S2, S3, S4, with their raw scores being 99+, 90, 89+ and 80 respectively, thus getting their grades to be A, A, B and B respectively. A small difference of 1% (90 for S2, 89 for S3) in the raw scores can result in a difference of 20% (5 for S2, 4 for S3) in grade points, because their raw scores happen to be across an interval boundary defined by the cut off marks of 90 between A and B. However, a large difference of almost 10% (9% between S1 & S2, and or between S3 & S4) does not result in any difference in their grades. Minor differences in raw scores get unduly amplified in the corresponding grades just because they are located across an interval boundary, while major differences in raw scores get unduly suppressed in the corresponding grades just because they are located within the very same interval. This undesirable system behavior gets further chaotic when the grades of various courses are combined to compute the GPA and/or CGPA, etc, as will be evident from the following discussion.

Now, consider a group of 22 students, with a wide range of academic performance levels, identified as 22S01, 22S02, . . . 22S22, each taking the six courses, of which three (s1/4c, s2/4c,s3/4c) are of 4 CCU, two (s4/3c, s5/3c) are of 3 CCU, and one (s6/2c) is of 2 CCU. Suppose that at the end of their academic term, their Raw Scores and Letter Grades in each of the courses are as listed in Table-1. An associated plot of sGPCCU versus sRSCCU is shown in Figure-1.

The Students Academic Performance Score (SAPS) is computed as follows: First, the sum (sRSCCU) of the raw scores, each of them being multipled by the corresponding Course Credits/Units, is computed. Then, sRSCCU is divided by the sum (sCCU) of Course Credits/Units (20, here) registered by the student for the semester; the ratio is the required SAPS value. To compute the Grade Point Average (GPA) first the sum (sGPCCU) of the Grade Points (associated with the Letter Grades) each of them being multiplied by the corresponding Course Credits/Units, is computed. Then, sGPCCU is divided by the sum (sCCU) of Course Credits/Units (20, here) registered by the student for the semester; the ratio is the required GPA value.

Table-1 gives, for each of the 22 students, for each of the six courses, the Raw Score (RS) and the Letter Grade (LG), along with sRSCCU, SAPS, SAPS-Rank, and also sQPCCU, GPA, GPA-Rank. The computations are as explained above.

The above computations do reveal some bare naked truth, about the lacunae or weaknesses or drawbacks in the prevailing grading system design. Specifically, the following observations are made.

(1) Phenomenon of Chaotic System Biased Amplification or Attenuation of Differentials (CSBAAD):

(a) Small difference in GPA associated with large difference in SAPS:
(22S01-22S03), (22S04-22S05), (22S06-22S08), (22S09-22S10),
(22S11-22S13), (22S14-22S15), (22S16-22S18), (22S19-22S20);

(b) Large difference in GPA associated with small difference in SAPS:
(22S02-22S06), (22S07-22S11), (22S12-22S16), (22S17-22S21);

(2) Phenomenon of Chaotic System Biased Suppression or Expression of Differentials (CSBSED):

(a) Large difference in SAPS associated with no difference in GPA:
(22S01-22S02), (22S06-22S07), (22S11-22S12),
(22S16-22S17), (22S21-22S22);

(b) No difference in SAPS associated with large difference in GPA:
(22S03-22S06), (22S08-22S11), (22S13-22S16), (22S18-22S21);

(3) Phenomenon of Chaotic System Biased Relative Rank Inversion (CSBRRI):

(a) Small decrease in GPA is associated with a large increase in SAPS:
(22S05-22S06), (22S10-22S11), (22S15-22S16), (22S20-22S21);

(b) Large decrease in GPA is associated a small increase in SAPS:
(22S02>22S03-22S04), (22S07>22S08-22S09),
(22S12>22S13-22S14), (22S17>22S18-22S19);

(c) As an even more drastic and shocking illustration of this phenomenon of *Chaotic System Biased Relative Rank Inversion (CSBRRI)*, observe that sometimes –
6 A's is no better than 1 A's and 5 B's : (22S02 < 22S04);
6 B's is no better than 1 B's and 5 C's : (22S07 < 22S09);
6 C's is no better than 1 C's and 5 D's : (22S12 < 22S14);
6 D's is no better than 1 D's and 5 E's : (22S17 < 22S19);
which correspond to the extreme instances of unfairness in the ubiquitous letter-grading-system.

One can observe from Figure-1 that the computation of GPA (and/or CGPA) yields results that lie within a finite closed region; that is, for each possible value of sGPCCU (or GPA or CGPA) there exists a finite closed interval of corresponding values of sRSCCU (or SAPS) and similarly for each possible value of sRSCCU (or SAPS) there exists a finite closed interval of corresponding values of sGPCCU (or GPA or CGPA).

As another example, consider another group of 38 students, with their academic performance levels (top performers in a class) being very close to one another, identified as 38S01, 38S02, . . . 38S38, each taking the six courses, of which three (s1/4c,s2/4c,s3/4c) are of 4 CCU, two (s4/3c,s5/3c) are of 3 CCU, and one (s6/2c) is of 2 CCU. Suppose that at the end of their academic term, their Raw Scores and Letter Grades in each of the courses are as listed in Table-2. An associated plot of sGPCCU versus sRSCCU is shown in Figure-2. Again it can be observed that the same chaotic system behavior is exhibited even in this situation. Although the variation in the GPA is only very minor, the corresponding variations in the SAPS can indeed be quite substantial. Also, the more drastic and shocking illustration of the phenomenon of *Chaotic System Biased Relative Rank Inversion (CSBRRI)*, can be observed, that sometimes –

6 A's and 0 B's is no better than 5 A's and 1 B's :
38S02 < 38S03; 38S02 < 38S05; 38S02 < 38S07;
6 A's and 0 B's is no better than 4 A's and 2 B's :
38S02 < 38S09; 38S02 < 38S11; 38S02 < 38S13; 38S02 < 38S15;
6 A's and 0 B's is no better than 3 A's and 3 B's :
38S02 < 38S17; 38S02 < 38S19; 38S02 < 38S21; 38S02 < 38S23;
6 A's and 0 B's is no better than 2 A's and 4 B's :
38S02 < 38S25; 38S02 < 38S27; 38S02 < 38S29;
6 A's and 0 B's is no better than 1 A's and 5 B's :
38S02 < 38S31; 38S02 < 38S33;
5 A's and 1 B's is no better than 4 A's and 2 B's :
38S04 < 38S09; 38S04 < 38S11; 38S04 < 38S13; 38S04 < 38S15;
5 A's and 1 B's is no better than 3 A's and 3 B's :
38S04 < 38S17; 38S04 < 38S19; 38S04 < 38S21; 38S04 < 38S23;
5 A's and 1 B's is no better than 2 A's and 4 B's :
38S04 < 38S25; 38S04 < 38S27; 38S04 < 38S29;
5 A's and 1 B's is no better than 1 A's and 5 B's :
38S04 < 38S31; 38S04 < 38S33; 38S04 < 38S35;
4 A's and 2 B's is no better than 3 A's and 3 B's :
38S10 < 38S17; 38S10 < 38S19; 38S10 < 38S21; 38S10 < 38S23;
4 A's and 2 B's is no better than 2 A's and 4 B's :
38S10 < 38S25; 38S10 < 38S27; 38S10 < 38S29;
4 A's and 2 B's is no better than 1 A's and 5 B's :
38S10 < 38S31; 38S10 < 38S33; 38S10 < 38S35;
4 A's and 2 B's is no better than 0 A's and 6 B's :
38S10 < 38S37;
3 A's and 3 B's is no better than 2 A's and 4 B's :
38S16 < 38S25; 38S16 < 38S27; 38S16 < 38S29;
3 A's and 3 B's is no better than 1 A's and 5 B's :
38S16 < 38S31; 38S16 < 38S33; 38S16 < 38S35;
3 A's and 3 B's is no better than 0 A's and 6 B's :
38S16 < 38S37;
2 A's and 4 B's is no better than 1 A's and 5 B's :
38S26 < 38S31; 38S26 < 38S33; 38S26 < 38S35;
2 A's and 4 B's is no better than 0 A's and 6 B's :
38S26 < 38S37;
1 A's and 5 B's is no better than 0 A's and 6 B's :
38S32 < 38S37.

It is to be noted here that the above observed chaotic system behavior is certainly not because of the specific choice of the set of various parameters and their values, but in fact an intrinsic characteristic of a poor system design, which happen to be exposed very well through these typical example scenarios.

The chaotic system behavior is neither arising from nor is avoidable by any teachers through their decisions on the evaluation itself. Also, such system behavior is neither associated with nor can be avoided by any general or specific students' action through their academic performance levels, whether individually or otherwise. The various parameters, like the number of courses considered for the analysis, the Course Credits/Units (CCU), the number of students being considered, the relative performance levels among them, the actual distribution of the raw scores in a class, and even the actual letter grading system model (policies and procedures) adopted, can in fact possibly be changed within their corresponding ranges of usually

acceptable variability, and still we can construct any number of example scenarios like the above, to illustrate such system behavior.

What do we conclude from the above observations? It is true that every teacher really means whatever s/he initially gives as the Raw Score for each of the test papers. All other transformations, conversions or computations, are only performed later, starting with these raw scores as the original raw input data; in order just to conform to the Policies and Procedures of the prevailing Evaluation System as per the Academic Rules and Regulations in the school/institute. However, the *"Student Academic Performance Score (SAPS)"* computed directly as a weighted average of the *original Raw Scores*, is certainly the best and in fact a true representation, of whatever the teachers really intended while originally evaluating each of their students.

Unfortunately, the prevailing LGS System just does not provide a reliable mechanism for a teacher to convey the information regarding the student academic performance evaluation, through the transcripts/grade reports, to whoever is really concerned in receiving (and possibly even acting, based on) such information! The system behavior exhibiting the *Phenomenon of Chaotic System Biased Amplification or Attenuation of Differentials (CSBAAD)*, the *Phenomenon of Chaotic System Biased Suppression or Expression of Differentials (CSBSED)*, and the *Phenomenon of Chaotic System Biased Relative Rank Inversion (CSBRRI)*, is certainly extremely undesirable, or rather just simply *unacceptable*.

The system is grossly *unfair to the students*, because they get subjected to *chaotic system biased unfair and unreliable comparisons* (CSBUUC), of course, not caused by any individual person, but because of the poor system design, which again is not a result of any deliberate intent of anyone. The system is *unfair to the recruiters* or prospective employers, because they are *misled by unreliable and/or misleading information*; when in fact, they deserve to be provided with the best and most reliable information on the students academic performance; by unbiased and fair measurements, with appropriately designed reliable performance indicators, and communicated through the transcripts or grade reports. The system is *unfair to the teachers* who although *helpless* (because of the existing system imposed on them, to be followed) are looked upon as the possible perpetrators of such a disturbingly chaotic system, which is both unfair as well as unreliable; wherein the Raw Scores/Marks that they had originally assigned, are later subjected to chaotic system biased information loss/corruption. Also, the LGS system design wrongly presumes that the teacher's precision in evaluation is rather poor, limited to classification into possibly only a handful of distinct categories. However, there is an intrinsic contradiction in the system design philosophy itself since a significantly higher precision level is mysteriously presumed to have been achieved in the final figures of GPA, CGPA, etc.

4. Concluding Remarks

The data presented here clearly shows the highly undesirable system behavior resulting in severe unfairness in the associated comparisons among the students; and that too not just among those in a small neighborhood of performance levels, but across the entire spectrum of performance levels. Further detailed analysis of such chaotic system characteristics will be presented as the second part of the report, in a later paper - *Academic Performance Evaluation of Students - Ubiquitous System Analyzed* (APES-USA) - clearly indicating that the root cause is some serious lacunae inherent in

its very design. The third part of the report "Student Academic Performance Evaluation System (SAPES) - A Proposed Solution to the APES-USA Problem" will also be presented in a later paper.

Student ID	Course No. / Credit Units Raw-Score | Letter-Grade												sGPCCU	GPA	GPA-Rank	sRSCCU	SAPS	SAPS-Rank
	s1 / 4c		s2 / 4c		s3 / 4c		s4 / 3c		s5 / 3c		s6 / 2c							
	RS	LG	RS	LG	RS	LG	RS	LG	RS	LG	RS	LG						
22S01	99	A	99	A	99	A	99	A	99	A	99	A	100	5	1	1980	99	1
22S02	90	A	90	A	90	A	90	A	90	A	90	A	100	5	1	1800	90	3
22S03	90	A	90	A	90	A	90	A	90	A	80	B	98	4.9	3	1780	89	4
22S04	99	A	89	B	89	B	89	B	89	B	89	B	84	4.2	4	1820	91	2
22S05	80	B	80	B	80	B	80	B	80	B	90	A	82	4.1	5	1620	81	6
22S06	89	B	89	B	89	B	89	B	89	B	89	B	80	4	6	1780	89	4
22S07	80	B	80	B	80	B	80	B	80	B	80	B	80	4	6	1600	80	8
22S08	80	B	80	B	80	B	80	B	80	B	70	C	78	3.9	8	1580	79	9
22S09	89	B	79	C	79	C	79	C	79	C	79	C	64	3.2	9	1620	81	6
22S10	70	C	70	C	70	C	70	C	70	C	80	B	62	3.1	10	1420	71	11
22S11	79	C	79	C	79	C	79	C	79	C	79	C	60	3	11	1580	79	9
22S12	70	C	70	C	70	C	70	C	70	C	70	C	60	3	11	1400	70	13
22S13	70	C	70	C	70	C	70	C	70	C	60	D	58	2.9	13	1380	69	14
22S14	79	C	69	D	69	D	69	D	69	D	69	D	44	2.2	14	1420	71	11
22S15	60	D	60	D	60	D	60	D	60	D	70	C	42	2.1	15	1220	61	16
22S16	69	D	69	D	69	D	69	D	69	D	69	D	40	2	16	1380	69	14
22S17	60	D	60	D	60	D	60	D	60	D	60	D	40	2	16	1200	60	18
22S18	60	D	60	D	60	D	60	D	60	D	50	E	38	1.9	18	1180	59	19
22S19	69	D	59	E	59	E	59	E	59	E	59	E	24	1.2	19	1220	61	16
22S20	50	E	50	E	50	E	50	E	50	E	60	D	22	1.1	20	1020	51	21
22S21	59	E	59	E	59	E	59	E	59	E	59	E	20	1	21	1180	59	19
22S22	50	E	50	E	50	E	50	E	50	E	50	E	20	1	21	1000	50	22

Table 1: A group of 22 students with wide-ranging performance levels.

Student ID	Course No. / Credit Units Raw-Score | Letter-Grade												sGPCCU	GPA	GPA-Rank	sRSCCU	SAPS	SAPS-Rank
	s1 / 4c		s2 / 4c		s3 / 4c		s4 / 3c		s5 / 3c		s6 / 2c							
	RS	LG	RS	LG	RS	LG	RS	LG	RS	LG	RS	LG						
38S01	99	A	99	A	99	A	99	A	99	A	99	A	100	5	1	1980	99	1
38S02	90	A	90	A	90	A	90	A	90	A	90	A	100	5	1	1800	90	18
38S03	99	A	99	A	99	A	99	A	99	A	89	B	98	4.9	3	1960	98	2
38S04	90	A	90	A	90	A	90	A	90	A	80	B	98	4.9	3	1780	89	20
38S05	99	A	99	A	99	A	99	A	89	B	99	A	97	4.85	5	1950	97.5	3
38S06	90	A	90	A	90	A	90	A	80	B	90	A	97	4.85	5	1770	88.5	22
38S07	99	A	99	A	89	B	99	A	99	A	99	A	96	4.8	7	1940	97	4
38S08	90	A	90	A	80	B	90	A	90	A	90	A	96	4.8	7	1760	88	23
38S09	99	A	99	A	99	A	99	A	89	B	89	B	95	4.75	9	1930	96.5	5
38S10	90	A	90	A	90	A	90	A	80	B	80	B	95	4.75	9	1750	87.5	24
38S11	99	A	99	A	89	B	99	A	99	A	89	B	94	4.7	11	1920	96	6
38S12	90	A	90	A	80	B	90	A	90	A	80	B	94	4.7	11	1740	87	25
38S13	89	B	99	A	99	A	99	A	89	B	99	A	93	4.65	13	1910	95.5	7
38S14	80	B	90	A	90	A	90	A	80	B	90	A	93	4.65	13	1730	86.5	26
38S15	99	A	99	A	99	A	89	B	89	B	89	B	92	4.6	15	1900	95	8
38S16	90	A	90	A	90	A	80	B	80	B	80	B	92	4.6	15	1720	86	27
38S17	99	A	99	A	89	B	99	A	89	B	89	B	91	4.55	17	1890	94.5	9
38S18	90	A	90	A	80	B	90	A	80	B	80	B	91	4.55	17	1710	85.5	28
38S19	99	A	99	A	89	B	89	B	89	B	99	A	90	4.5	19	1880	94	10
38S20	90	A	90	A	80	B	80	B	80	B	90	A	90	4.5	19	1700	85	29
38S21	99	A	89	B	89	B	89	B	99	A	99	A	89	4.45	21	1870	93.5	11
38S22	90	A	80	B	80	B	80	B	90	A	90	A	89	4.45	21	1690	84.5	30
38S23	89	B	89	B	89	B	99	A	99	A	99	A	88	4.4	23	1860	93	12
38S24	80	B	80	B	80	B	90	A	90	A	90	A	88	4.4	23	1680	84	31
38S25	99	A	89	B	89	B	99	A	89	B	89	B	87	4.35	25	1850	92.5	13
38S26	90	A	80	B	80	B	90	A	80	B	80	B	87	4.35	25	1670	83.5	32
38S27	89	B	89	B	89	B	99	A	99	A	89	B	86	4.3	27	1840	92	14
38S28	80	B	80	B	80	B	90	A	90	A	80	B	86	4.3	27	1660	83	33
38S29	89	B	89	B	89	B	89	B	99	A	99	A	85	4.25	29	1830	91.5	15
38S30	80	B	80	B	80	B	80	B	90	A	90	A	85	4.25	29	1650	82.5	34
38S31	99	A	89	B	89	B	89	B	89	B	89	B	84	4.2	31	1820	91	16
38S32	90	A	80	B	80	B	80	B	80	B	80	B	84	4.2	31	1640	82	35
38S33	89	B	89	B	89	B	99	A	89	B	89	B	83	4.15	33	1810	90.5	17
38S34	80	B	80	B	80	B	90	A	80	B	80	B	83	4.15	33	1630	81.5	36
38S35	89	B	89	B	89	B	89	B	89	B	99	A	82	4.1	35	1800	90	18
38S36	80	B	80	B	80	B	80	B	80	B	90	A	82	4.1	35	1620	81	37
38S37	89	B	89	B	89	B	89	B	89	B	89	B	80	4	37	1780	89	20
38S38	80	B	80	B	80	B	80	B	80	B	80	B	80	4	37	1600	80	38

Table-2. A group of 38 students in the top performance band.

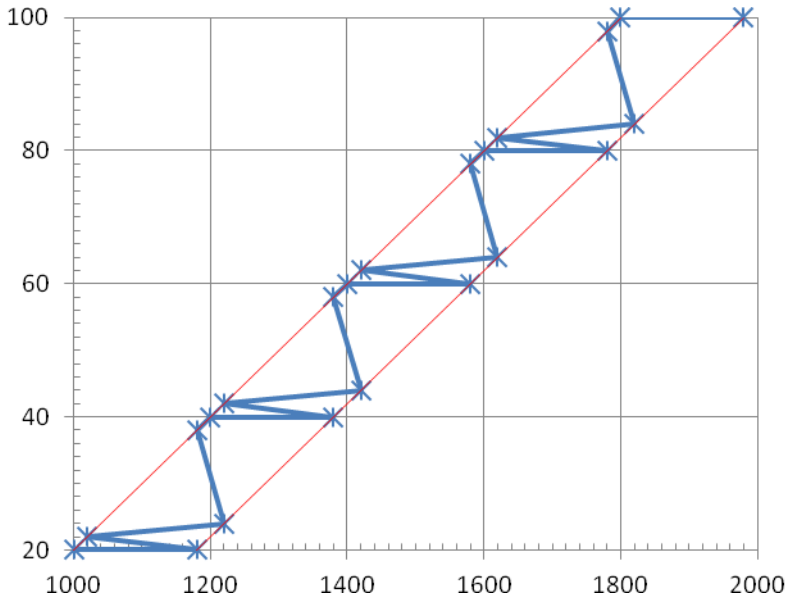

Figure-1: Plot of sGPCCU versus sRSCCU for 22 students.

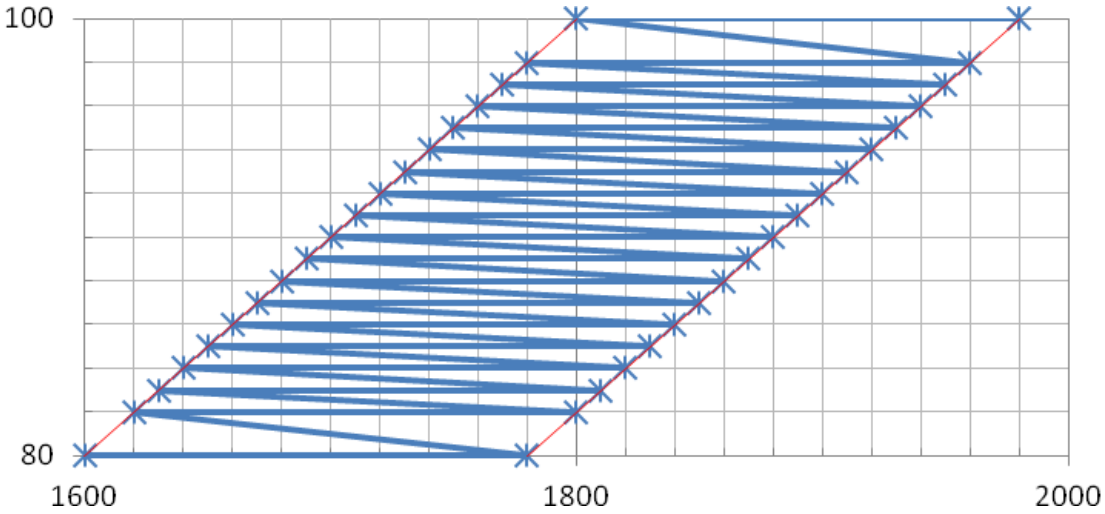

Figure-2: Plot of sGPCCU versus sRSCCU for 38 students.